Variable Modified Chaplygin Gas in Anisotropic Universe with Kaluza-Klein Metric

Chayan Ranjit[1*] **Shuvendu Chakraborty**[2†] **and Ujjal Debnath**[3‡]
[1,2]*Department of Mathematics, Seacom Engineering College, Howrah - 711 302, India.*
[3]*Department of Mathematics, Bengal Engineering and Science University, Shibpur, Howrah-711 103, India.*

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In this work, we have consider Kaluza-Klein Cosmology for anisotropic universe where the universe is filled with variable modified chaplygin gas (VMCG). Here we find normal scalar field ϕ and the self interacting potential $V(\phi)$ to describe the VMCG Cosmology. Also we graphically analyzed the geometrical parameters named *statefinder parameters* in anisotropic Kaluza-Klein model. Next, we consider a Kaluza-Klein model of interacting VMCG with dark matter in the Einstein gravity framework. Here we construct the three dimensional autonomous dynamical system of equations for this interacting model with the assumption that the dark energy and the dark matter are interact between them and for that we also choose the interaction term. We convert that interaction terms to its dimensionless form and perform stability analysis and solve them numerically. We obtain a stable scaling solution of the equations in Kaluza-Klein model and graphically represent solutions.

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I. INTRODUCTION

The recent observations of the luminosity of type Ia supernovae [1, 2] indicate that the universe is currently undergoing an accelerated expansion. The theory of Dark energy having negative pressure is the main responsible for this scenario. From recent cosmological observations including supernova data [3] and measurements of cosmic microwave background radiation (CMBR) [4] it is evident that our present universe is made up of about 2/3 dark energy and about 1/3 dark matter of the whole universe. There are several interesting mechanisms such as higher dimensional phenomena [5], Loop Quantum Cosmology (LQC) [6], modified gravity [7], Brans-Dicke theory [8], brane-world model [9] and many others which are extremely applicable in recent research to explain the present scenario of the universe.

A fundamental theory on higher dimensional model was introduced by Kaluza-Klein [10, 11], where they considered an extra dimension with FRW metric to unify Maxwell's theory of electromagnetism and Einstein's gravity. After extensive research, scientists are realized that higher dimensional Cosmology may be more useful to understand the interaction of particles. Also to study the universe, as our space-time is explicitly four dimensional in nature, the 'hidden' dimensions must be related to the dark matter and dark energy. In modern physics Kaluza-Klein theory showed its great importance in string theory [12], in supergravity [13]and in superstring theories [14]. Li et al [15] have considered the inflation in Kaluza-Klein theory. In 1997, Overduin and Wesson [16] represented a review of Kaluza-Klein theory with higher dimensional unified theories. String cloud and domain walls with quark matter in N-dimensional Kaluza-Klein Cosmological model was presented by Adhav et al [17]. Recently some authors [18–24] studied Kaluza-Klein cosmological models with different dark energies and dark matters.

Recently dynamical system theory become a very important tool to study cosmology and astrophysics in the framework of general relativity. Some authors [26–30] work on qualitative analysis in dynamical system of FRW Cosmology in Brans-Dicke gravity, where they showed that the field equations of FRW may be reduced to two dimensional dynamical system whether in presence of scalar potential it could be reduced in three dimensional.

In this work, we have considered Kaluza-Klein Cosmology for Anisotropic Universe where the Universe is filled with variable modified Chaplygin gas (VMCG). Here we find normal scalar field ϕ and the self

* chayanranjit@gmail.com
† shuvendu.chakraborty@gmail.com
‡ ujjaldebnath@yahoo.com, ujjal@iucaa.ernet.in

interacting potential $V(\phi)$ to describe the VMCG Cosmology. Also we graphically analyzed the geometrical parameters named *statefinder parameters* in anisotropic Kaluza-Klein model. In the later section, we consider a Kaluza-Klein model of interacting VMCG with dark matter in the Einstein gravity framework. Here we construct the three dimensional autonomous dynamical system of equations for this interacting model with the assumption that the dark energy and the dark matter are interact between them and for that we also choose the interaction term. We convert that interaction terms to its dimensionless form and perform stability analysis and solve them numerically. We obtain a stable scaling solution of the equations in Kaluza-Klein model and graphically represent solutions.

II. BASIC EQUATIONS AND SOLUTIONS

The Kaluza-Klein type metric is given by [25]

$$ds^2 = dt^2 - a^2(dx^2 + dy^2 + dz^2) - b^2 d\psi^2 \tag{1}$$

where a and b are function of cosmic time t only. Here we are dealing only with an anisotropic fluid whose energy momentum tensor is in the following form:

$$T^u_v = diag[T^0_0, T^1_1, T^3_3, T^4_4]$$

We parameterize it as follows:

$$T^u_v = diag[\rho, -p_x, -p_y, -p_z, -p_\psi]$$

$$= diag[1, -\omega_x, -\omega_y, -\omega_z, -\omega_\psi]\rho$$

$$= diag[1, -\omega, -\omega, -\omega, -\omega]\rho \tag{2}$$

where ρ is the energy density of the fluid; p_x, p_y, p_z and p_ψ are the pressures and $\omega_x, \omega_y, \omega_z$ and ω_ψ are the directional equation of state (EoS) parameters of the fluid.

The Einstein field equations, in natural limits ($8\pi G = c = 1$) are

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = -T_{\mu\nu} \tag{3}$$

where $g_{\mu\nu}u^\mu u^\nu = 1; u^\mu = (1, 0, 0, 0, 0)$ is the velocity vector; $R_{\mu\nu}$ is the Ricci tensor; R is the Ricci scalar, $T_{\mu\nu}$ is the energy momentum tensor. In a co-moving coordinate system, Einstein's filled equation (3), for the anisotropic Kaluza-Klein spacetime (1), with eq.(2) (for flat universe i.e., for $k = 0$) yield

$$3\frac{\dot{a}\dot{b}}{ab} + 3\frac{\dot{a}^2}{a^2} = \rho \tag{4}$$

$$2\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}\dot{b}}{ab} = -p \tag{5}$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} = -p \tag{6}$$

where the overhead dot($\dot{}$) denote derivative with respect to the cosmic time t. The energy conservation equation in Kaluza-Klein Cosmological Model is as

$$\dot{\rho} + 4H(\rho + p) = 0 \tag{7}$$

In the case of *pure Chaplygin gas* obeys an equation of state like [31, 32]

$$p = -\frac{B}{\rho} \tag{8}$$

where p and ρ are respectively the pressure and energy density and B is a positive constant. Subsequently in the case of *generalized Chaplygin gas* [33–35] obeys an equation of state

$$p = -\frac{B}{\rho^{\alpha}} \qquad with \quad 0 \leq \alpha \leq 1 \tag{9}$$

and in the case of *modified Chaplygin gas* [36, 37] obeys an equation of state

$$p = A\rho - \frac{B}{\rho^{\alpha}} \qquad with \quad 0 \leq \alpha \leq 1 \tag{10}$$

where A, B are positive constants. This equation of state shows that radiation era (when $A = 1/3$) at one extreme (when the scale factor $a(t)$ is vanishingly small) while a ΛCDM model at other extreme (when the scale factor $a(t)$ is infinitely large).

Guo and Zhang 2005*a* [38] first proposed *variable Chaplygin gas* model with equation of state (8), where B is a positive function of the cosmological scale factor 'a' i.e., $B = B(a)$. This assumption is reasonable since $B(a)$ is related to the scalar potential if we take the Chaplygin gas as a Born- Infeld scalar field *Bento et al. 2003* [39]. There are some works on variable Chaplygin gas model like as *Sethi et al.* 2005 [40], *Guo and Zhang* 2005*b* [41]. In 2007, *Debnath* [42] introduced VMCG with equation of state (10) where B is a positive function of the cosmological scale factor 'a' ($i.e., B = B(a)$) as

$$p = A\rho - \frac{B(a)}{\rho^{\alpha}} \qquad with \quad 0 \leq \alpha \leq 1 \tag{11}$$

where A is a positive constant. Now, we have considered that the anisotropic universe filled with VMCG obeys equation of state (10) where B is a positive function of the cosmological scale factors a and b (i.e., $B = B(a, b)$)

$$p = A\rho - \frac{B(a,b)}{\rho^{\alpha}} \qquad with \quad 0 \leq \alpha \leq 1 \tag{12}$$

Now, for simplicity, assume $B(a, b)$ is of the form

$$B(a,b) = B_0 \left(a^3 b\right)^{-n} \tag{13}$$

where $B_0 > 0$ and n are constants. From (7) using (12) and (13) we have

$$\rho = \left[\frac{B_0(\alpha+1)}{((A+1)(\alpha+1)-n)}\frac{1}{(a^3b)^n} + \frac{C}{(a^3b)^{(\alpha+1)(A+1)}}\right]^{\frac{1}{\alpha+1}} \tag{14}$$

where $C > 0$ is an arbitrary integration constant and $(1+A)(1+\alpha) > n$, for positivity of first term. Here n must be positive, because otherwise, $a \to \infty$ implies $\rho \to \infty$, which is not the case for expanding Universe. Now, we consider the energy density and pressure corresponding to normal scalar field ϕ having a self-interacting potential $V(\phi)$ as follows:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \rho = \left[\frac{B_0(\alpha+1)}{((A+1)(\alpha+1)-n)}\frac{1}{(a^3b)^n} + \frac{C}{(a^3b)^{(\alpha+1)(A+1)}}\right]^{\frac{1}{\alpha+1}} \tag{15}$$

and

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = A\rho - \frac{B_0(a^3b)^{-n}}{\rho^{\alpha}} = A\left[\frac{B_0(\alpha+1)}{((A+1)(\alpha+1)-n)}\frac{1}{(a^3b)^n} + \frac{C}{(a^3b)^{(\alpha+1)(A+1)}}\right]^{\frac{1}{\alpha+1}}$$
$$- \frac{B_0(a^3b)^{-n}}{\left[\frac{B_0(\alpha+1)}{((A+1)(\alpha+1)-n)}\frac{1}{(a^3b)^n} + \frac{C}{(a^3b)^{(\alpha+1)(A+1)}}\right]^{\frac{\alpha}{\alpha+1}}} \tag{16}$$

Hence for flat universe ($i.e., k = 0$), we have

$$\phi = \int \left[(A+1)\left(\frac{B_0(\alpha+1)(a^3b)^{-n}}{1+A-n+\alpha+A\alpha} + C\left(a^3b\right)^{-(A+1)(\alpha+1)}\right)^{\frac{1}{\alpha+1}}\right.$$

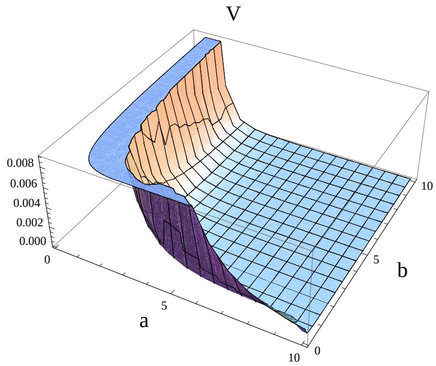

Fig.1

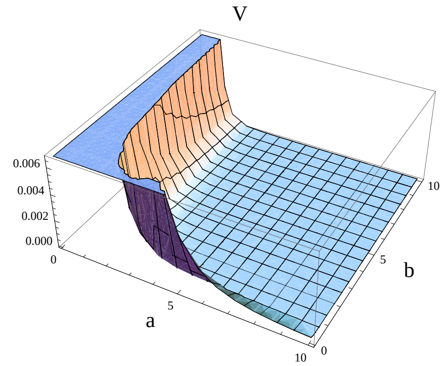

Fig.2

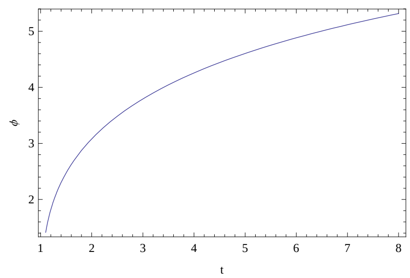

Fig.3

Fig. 1 and Fig. 2 show the variations of V against a and b for $A = \frac{1}{3}$ and $A = 1$ respectively with $B = 1, C = 2, n = 3, \alpha = 0.8$ and Fig. 3 shows the variation of ϕ against t for $A = \frac{1}{3}, B = 2, C = 1, n = 1.1, d = 1, \alpha = 0.5$ in the case of the anisotropic universe filled with *Variable Modified Chaplygin Gas* with Kaluza-Klein metric.

$$-B_0(a^3b)^{-n}\left(\left(\frac{B_0(\alpha+1)(a^3b)^{-n}}{1+A-n+\alpha+A\alpha}+C\left(a^3b\right)^{-(A+1)(\alpha+1)}\right)^{\frac{1}{\alpha+1}}\right)^{-\alpha}\Bigg]^{\frac{1}{2}}\,dt \tag{17}$$

and

$$V=\frac{1}{2}\Bigg[-(A-1)\left(\frac{B_0(\alpha+1)(a^3b)^{-n}}{1+A-n+\alpha+A\alpha}+C\left(a^3b\right)^{-(A+1)(\alpha+1)}\right)^{\frac{1}{\alpha+1}}$$

$$-B_0(a^3b)^{-n}\left(\left(\frac{B_0(\alpha+1)(a^3b)^{-n}}{1+A-n+\alpha+A\alpha}+C\left(a^3b\right)^{-(A+1)(\alpha+1)}\right)^{\frac{1}{\alpha+1}}\right)^{-\alpha}\Bigg] \tag{18}$$

From equation (17), we see that the integration is very complicated. So the numerical investigations are needed to get the physical interpretation of the scalar field and the corresponding potential. Fig. 1 and Fig. 2 show the variations of V against a and b for $A = \frac{1}{3}$ and $A = 1$ respectively with $B = 1, C = 2, n = 3, \alpha = 0.8$ and Fig. 3 shows the variation of ϕ against t for $A = \frac{1}{3}, B = 2, C = 1, n = 1.1, d = 1, \alpha = 0.5$ in the case of the anisotropic universe filled with *Variable Modified Chaplygin Gas* with Kaluza-Klein metric. From the figures, we see that ϕ increases due to evolution of the universe. Also the potential V decreases with the scale factors a and b.

In the paper Gorini et al. 2003 [33]; Alam et al. 2003 [34], the flat Friedmann model filled with Chaplygin fluid has been analyzed in terms of the recently proposed statefinder parameters Sahni et al. 2003 [43]. The statefinder diagnostic along with future SNAP observations may perhaps be used to discriminate between

different dark energy models. The above statefinder diagnostic pair for higher dimensional anisotropic cosmology are constructed from the scale factors a and b as follows:

$$r = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \quad \text{and} \quad s = \frac{r-1}{3(q-\frac{1}{2})} \tag{19}$$

where q is the deceleration parameter defined by $q = -1 - \frac{\dot{H}}{H^2}$ and H is the Hubble parameter. Since this parameters are dimensionless so they allow us to characterize the properties of dark energy in a model independently. Finally we graphically analyzed geometrical parameters r and s with respect to a and b in the case of the anisotropic universe filled with *Variable Modified Chaplygin Gas*.

For Kaluza-Klein model with flat universe ($i.e., k = 0$) we have

$$4\dot{H} + 16H^2 = \frac{4}{3}(\rho - p) \tag{20}$$

Therefore

$$q = 3 - \frac{\rho - p}{3H^2} \tag{21}$$

Hence

$$r = 21 + \frac{4(\rho - p)}{3H^2}\left[\frac{7}{4} + (\alpha+1)A - \frac{\alpha p}{\rho}\right] + \frac{4n}{3H^2}(p - A\rho) \tag{22}$$

and

$$s = \frac{4}{3(\frac{15}{2} - \frac{p-\rho}{H^2})}\left[15 + \frac{\rho - p}{H^2}(\frac{7}{4} + (\alpha+1)A - \frac{\alpha p}{\rho}) + \frac{n}{H^2}(p - A\rho)\right] \tag{23}$$

where ρ and p are given by (15) and (16). Using field equations, r and s can be expressed in terms of scale factors a and b in very complicated form, so we have not presented the explicit expressions here. Fig. 4 and Fig. 5 show the variations of r against a and b and the variations of s against a and b and Fig. 6 shows the variation of r against s for $A = 1, B = 1, C = 2, n = 3, \alpha = 0.8$ and $H = 1$ respectively in the case of the anisotropic universe filled with *Variable Modified Chaplygin Gas*. From the figures, we conclude that r increases and s decreases as a and b increase. From figure 6, we see that s increases from some positive value to $+\infty$ as r decreases from finite positive range. Also after certain stage, s increases from $-\infty$ to 0 as r decreases from some positive to negative value.

III. DYNAMICAL SYSTEM ANALYSIS IN KALUZA-KLEIN MODEL WITH INTERACTING VMCG AND DARK MATTER

We consider the universe filled with VMCG as dark energy and dark matter interacting through an interaction term. Due to interaction between the two components, the energy conservation would not hold for the individual components, therefore the above conservation equation (7) will break into two non-conserving equations:

$$\dot{\rho}_{vmcg} + 4H(\rho_{vmcg} + p_{vmcg}) = -Q \tag{24}$$

and

$$\dot{\rho}_{dm} + 4H(\rho_{dm} + p_{dm}) = Q \tag{25}$$

Here Q is the interaction term which have dimension of density multiplied by Hubble parameter H. For this scenario the suitable choice of Q be $Q = 4bH\rho$, b is the coupling parameter denoting the transfer strength, $\rho = \rho_{vmcg} + \rho_{dm}$ and $p = p_{vmcg} + p_{dm}$ (p_{dm} is very small quantity, somewhere dark matter assumed as pressureless quantity) are the total cosmic energy density and pressure respectively which satisfy the above

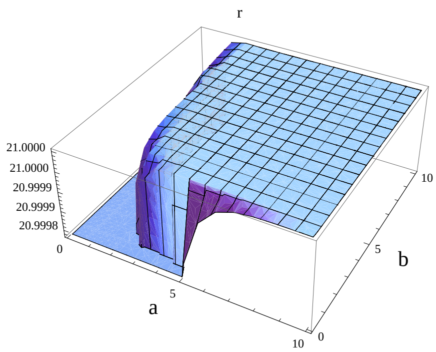

Fig.4

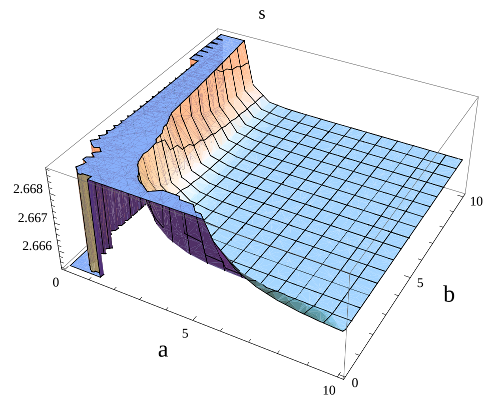

Fig.5

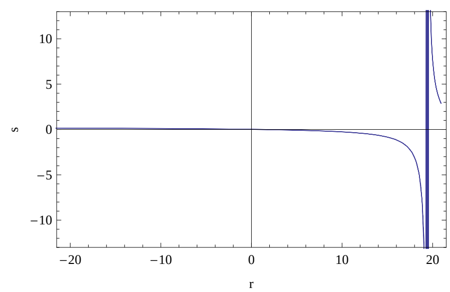

Fig.6

Fig. 4 and Fig. 5 show the variations of r against a and b and the variations of s against a and b for $A = 1, B = 1, C = 2, n = 3, \alpha = 0.8$ and $H = 1$ respectively and Fig. 6 shows the variation of r against s for $A = 1, B = 1, C = 2, n = 3, \alpha = 0.8$ and $H = 1$ respectively in the case of the anisotropic universe filled with *Variable Modified Chaplygin Gas*.

conservation equation (7). The subscripts $vmcg$ and dm denote the VMCG and dark matter respectively.

The dark matter equation of state is assumed to be linear EoS

$$p_{dm} = \omega_{dm}\rho_{dm} \tag{26}$$

To analyze the dynamical system, we convert the physical parameter into dimensionless form as follows:

$$x = \ln(a^3 b), \qquad u = \frac{\rho_{vmcg}}{4H^2}, \qquad v = \frac{p_{vmcg}}{4H^2}, \qquad w = \frac{\rho_{dm}}{4H^2} \tag{27}$$

The equation of state of the VMCG can be expressed as

$$\omega_{vmcg} = \frac{p_{vmcg}}{\rho_{vmcg}} = \frac{v}{u} \tag{28}$$

Now we can cast the evolution equations in the following autonomous system of u, v and w in the form:

$$\frac{du}{dx} = u - v - cu - cw - \frac{2}{3}u(u - v + (1 - \omega_{dm})w) \tag{29}$$

$$\frac{dv}{dx} = ((1+\alpha)A - \alpha\frac{v}{u})(-u-v-c(u+w)) + n(Au-v) - \frac{2}{3}v(u-v+(1-\omega_{dm})w) + 2v \tag{30}$$

$$\frac{dw}{dx} = -(1+\omega_{dm})w + c(u+w) - \frac{2}{3}w(u-v+(1-\omega_{dm})w) + 2w \tag{31}$$

For the mathematical simplicity, we work out $\alpha = 1$ and $A = \frac{1}{3}$ only. The critical points of the above system are obtained by putting $\frac{du}{dx} = \frac{dv}{dx} = \frac{dw}{dx} = 0$ which yield

$$u_{crit} = \frac{3}{2} + \frac{3c}{n - 2(1+\omega_{dm})} \tag{32}$$

$$v_{crit} = \frac{3(n^2 - 2n(2+\omega_{dm}) + 4(1+\omega_{dm}+c\omega_{dm}))}{4(n-2(1+\omega_{dm}))} \tag{33}$$

$$w_{crit} = \frac{3c}{2 - n + 2\omega_{dm}} \tag{34}$$

For the stability of the dynamical system about the critical point, we linearize the governing equation around the critical point and arrived at

$$\delta\left(\frac{du}{dx}\right) = \left[\frac{1}{3}\left(3 - 3c - 4u + 2v + 2w(-1+\omega_{dm})\right)\right]_{crit}\delta u + \left[\frac{2u}{3} - 1\right]_{crit}\delta v + \left[\frac{1}{3}\left(-3c + 2u(-1+\omega_{dm})\right)\right]_{crit}\delta w \tag{35}$$

$$\delta\left(\frac{dv}{dx}\right) = \left[-\frac{3Au^2(1+c-n+\alpha+c\alpha) + v(2u^2+3(v+cw)\alpha)}{3u^2}\right]_{crit}\delta u + \left[2 - n - \frac{2u}{3} + \frac{4v}{3} + (1+c)\alpha\right.$$

$$\left. + \frac{(2v+cw)\alpha}{u} - A(1+\alpha) + \frac{2}{3}w(-1+\omega_{dm})\right]_{crit}\delta v + \left[-Ac(1+\alpha) + \frac{1}{3}v(-2 + \frac{3c\alpha}{u} + 2\omega_{dm})\right]_{crit}\delta w \tag{36}$$

$$\delta\left(\frac{dw}{dx}\right) = \left[c - \frac{2w}{3}\right]_{crit}\delta u + \left[\frac{2w}{3}\right]_{crit}\delta v + \left[\frac{1}{3}(3c - 2u + 2v + (-3+4w)(-1+\omega_{dm}))\right]\delta w \tag{37}$$

The corresponding eigenvalues of the coefficient matrix of the above equations are

$$\lambda_1 = \frac{n-4}{2} \tag{38}$$

and

$$\lambda_{2,3} = \frac{44 - 40n + 9n^2 + 4c(3n-8) + 56\omega_{dm} - 24n\omega_{dm} + 12\omega_{dm}^2}{12(2c+n-2(1+\omega_{dm}))}$$

$$\pm\frac{\sqrt{(8c(3\omega_{dm}-1) + (6\omega_{dm}+3n-10)(n-2(\omega_{dm}+1)))(-48c^2 + 8c(9\omega_{dm}-3n+5) + (6\omega_{dm}+3n-10)(n-2(\omega_{dm}+1)))}}{12(2c+n-2(1+\omega_{dm}))} \tag{39}$$

If the real parts of the above eigenvalues are negative at critical points then critical points are stable and are stationary attractor solutions; otherwise unstable and thus oscillatory. Therefore the physical meaningful ranges are $n < \frac{8}{3}$; $c > 0$ and $\frac{-3n^2+14n+4c-16}{-6n+12c+16} < \omega_{dm} \leq \frac{1}{3}(3c+1) - \frac{\sqrt{9n^2-48n+36c^2+64}}{6}$ and in this range the critical point $(u_{crit}, v_{crit}, w_{crit})$ is stable and is a late-time stationary attractor solution.

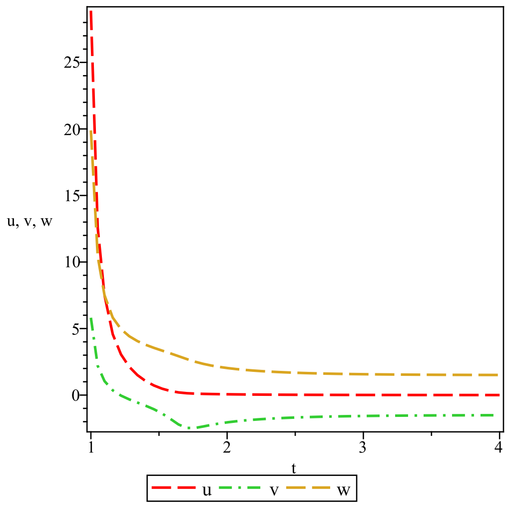

Fig.7

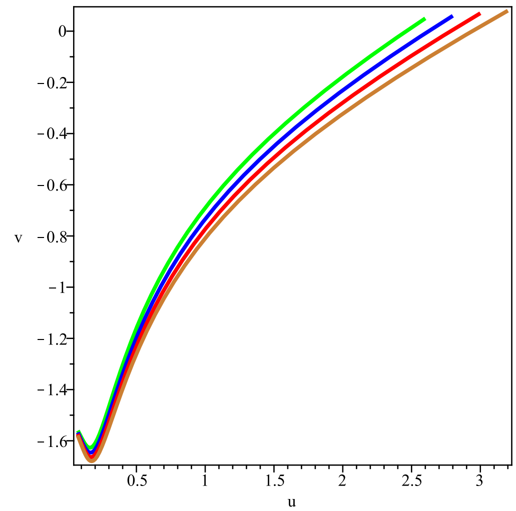

Fig.8

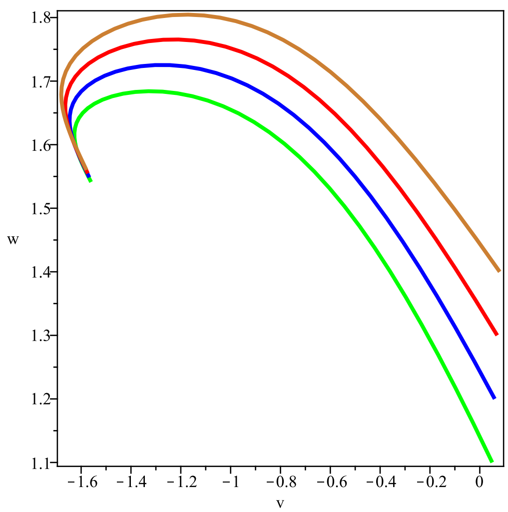

Fig.9

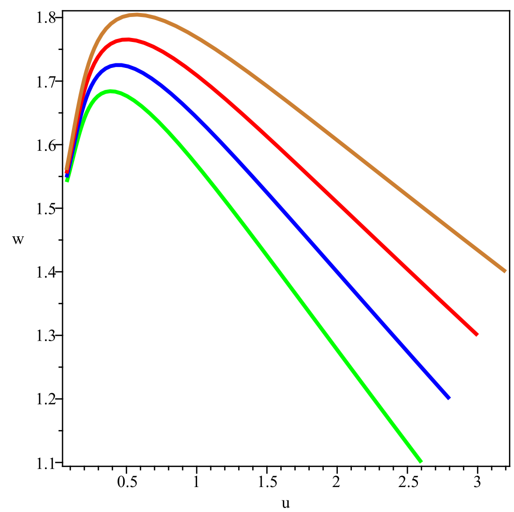

Fig.10

Fig. 7 shows the variations of the dimensionless parameters u,v and w against the time for the initial conditions $u(1.1) = 2.5, v(1.1) = 0.05$, and $w(1.1) = 2.8$. Fig. 8,9,10 show the phase space diagram in the sense of $[v(t), u(t)]$,$[w(t), v(t)]$ and $[w(t), u(t)]$ respectively.

The dimensionless parameters u, v, w are drawn in figure 7 and we see that these are decreasing in progression of time, whereas u, w keep positive orientation and v keeps negative sign in late time evolution. Also v against u, w against v and w against u are drawn in figures 8, 9 and 10 respectively. Here v keeps negative sign when u and w increase. The 3D figure of u, v, w are drawn in figures 11 and 12 to get the visualization of the natures of u, v and w for 4 different initial conditions and a single initial condition respectively. The deceleration parameter q from figure 13 shows that q decreases from .5 to -1 which interprets the early deceleration and the late time acceleration generated by the VMCG in Kaluza-Klein Cosmology.

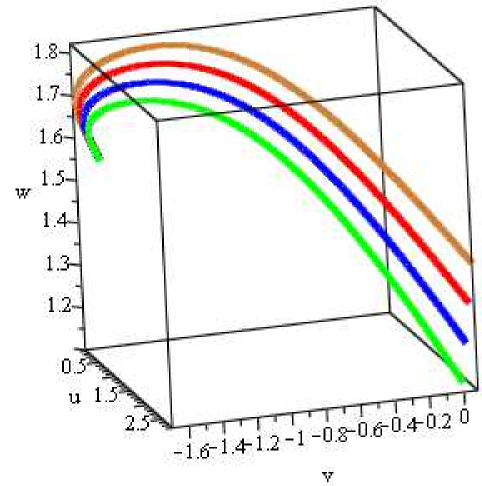

Fig.11

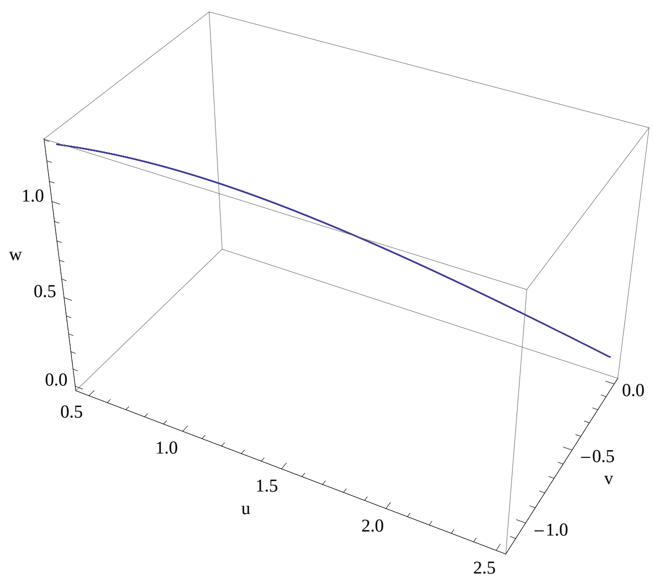

Fig.12

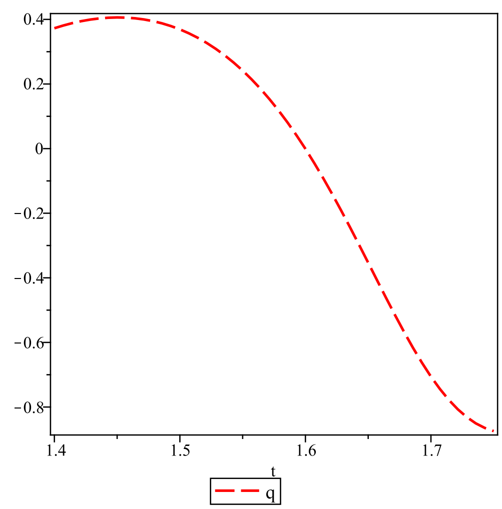

Fig.13

Fig. 11 and Fig. 12 show the 3D variations of the dimensionless parameters u,v and w for four different initial conditions and single initial condition. Fig. 13 shows the variation of deceleration parameter q against the time t.

IV. CONCLUSIONS

In this work, we have assumed the 5D Kaluza-Klein Cosmology for anisotropic universe where the universe is filled with variable modified Chaplygin gas (VMCG). The function B is assumed as a function of the scale factors a and b with some suitable choice $B = B_0 \left(a^3 b\right)^{-n}$. If the energy density and the pressure corresponding to the normal scalar field ϕ and the self interacting potential $V(\phi)$ can be associated with the energy density

and the pressure of the VMCG, the corresponding ϕ and V have been calculated in terms of the scale factors a and b. The variations of V against a and b are drawn in figures 1 and 2 and have seen that V decreases as universe expands. Also from figure 3, we see that ϕ always increases as time goes on. Also we graphically analyzed the geometrical parameters named *statefinder parameters* in anisotropic Kaluza-Klein model. Fig. 4 and Fig. 5 show the variations of r against a and b and the variations of s against a and b and Fig. 6 shows the variation of r against s respectively for some particular choices of the constants. From the figures, we conclude that r increases and s decreases as a and b increase. From figure 6, we see that s increases from some positive value to $+\infty$ as r decreases from finite positive range. Also after certain stage, s increases from $-\infty$ to 0 as r decreases from some positive to negative value.

Next, we consider a Kaluza-Klein model of interacting VMCG with dark matter in the framework of Einstein gravity. Here we construct the three dimensional autonomous dynamical system of equations for this interacting model with the assumption that the dark energy and the dark matter are interact between them and for that we also choose the interaction term Q in the form $4bH\rho$, which is of course decreasing with the expansion of the universe. We convert that interaction terms to its dimensionless form and perform stability analysis and solve them numerically. The critical point $(u_{crit}, v_{crit}, w_{crit})$ have been found for the autonomous system. We obtain a stable scaling solution of the equations in Kaluza-Klein model and graphically represent solutions and have shown that the critical point is a late time stable attractor. The dimensionless parameters u, v, w are drawn in figure 7 and we see that these are decreasing in progression of time, whereas u, w keep positive orientation and v keeps negative sign in late time evolution. Also v against u, w against v and w against u are drawn in figures 8, 9 and 10 respectively. Here v keeps negative sign when u and w increase. The 3D figure of u, v, w are drawn in figures 11 and 12 to get the visualization of the natures of u, v and w for 4 different initial conditions and a single initial condition respectively. The deceleration parameter q from figure 13 shows that q decreases from .5 to -1 which interprets the early deceleration and the late time acceleration generated by the VMCG in Kaluza-Klein Cosmology.

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